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Technical Note

Drag reduction and heat transfer enhancement over a heated wall of a vertical annular microchannel

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ABSTRACT

An analysis for the effect of wall-surface curvature on gas microflow is performed to study the natural convection in an open-ended vertical annular microchannel with an isothermally heated inside wall. The fully developed solutions of the velocity, temperature, flow rate, shear stress, and heat flux are derived analytically and presented for air and various surfaces at the standard reference state. Results show that wall-surface curvature has a significant effect. This results in a nonlinear behavior in the temperature, which seems difficult to appear in a parallel-plate microchannel. Under certain rarefaction and fluid–wall interaction conditions, by decreasing the value of the curvature radius ratio, it is possible to obtain both reduced flow drag and enhanced heat transfer.

1. Introduction

Microelectromechanical systems and micrototal analysis systems develop a large number of microfluidic systems in silicon, quartz, glass, plastics, etc. In these systems, rarefied gas flow can be observed [1]; moreover, fluid–wall interaction, which is related to the material and quality of channel wall, i.e., its type and roughness, plays an important role in the momentum and energy exchanges [2]. Thus, the velocity slip and temperature-jump conditions induced by the effects of rarefaction and fluid–wall interaction should be considered in convective gas microflow problems. In addition, the technological development for reducing flow drag and enhancing heat transfer could have an enormous economic impact as microelectromechanical systems are miniaturized and microfluidic system devices become more widely used. The phenomena of drag reduction and heat transfer enhancement should, therefore, be stressed in theoretical and experimental work.

Natural convective gas microflow, encountered in many engineering fields, e.g., microelectrochemical cell transport, microheat exchanging, and microchip cooling, is an attractive branch of microfluidics [3] due to its reliability, simplicity, and cost effectiveness in flow and heat transfer mechanism. Earlier work was analytically studied by Chen and Weng [4] for the fully developed natural convection in a vertical parallel-plate microchannel with asymmetric wall temperature distributions. The effects of rarefaction and fluid–wall interaction were shown to increase the flow rate and to decrease the heat flux. Khadrawi et al. [5] analytically investigated the transient hydrodynamics and thermal behaviors under the effect of the hyperbolic heat conduction model. Haddad et al. [6] reported implicit finite-difference simulations of the developing natural convection in an isothermally heated microchannel filled with porous media. Biswal et al. [7] investigated the flow and heat transfer characteristics in the developing region of a microchannel by using the semi-implicit method for pressure linked equations (SIMPLE) and concluded that the microscale effects associated with the velocity slip and the temperature jump enhance the heat transfer. Chen and Weng [8] emphasized the importance of thermal creep and high-order slip/jump in developing natural convection by using a marching implicit (MI) procedure. Chakraborty et al. [9] further executed a boundary layer integral analysis to investigate the heat transfer characteristics under large channel aspect ratios. Recently, Avci and Aydin [10] analytically investigated the effect of pressure gradient on the gas microflow obtained by Chen and Weng, the so-called mixed convective gas microflow. Avci and Aydin [11] further extended their study to the case where constant wall heat fluxes are considered. Weng and Chen [12] placed special emphasis on the importance of thermal creep in natural convective gas microflow with constant wall heat fluxes.

Microfluidic system devices with curved channel surfaces are frequently encountered. One of the basic steps in investigating the effect of wall-surface curvature is to study the flow in an annular microchannel [13–15]. In this work, the fully developed natural convective gas flow in an open-ended vertical annular microchannel

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with an isothermally heated inside wall is studied. Navier–Stokes (NS) equations subject to first-order slip/jump boundary conditions are analytically solved. The purpose is to examine the influence of wall-surface curvature on the flow and thermal fields as well as the corresponding characteristics over the heated wall. Such a type of study may be used on the determination of the thermal and tangential momentum accommodation coefficients [16] and be applicable to the designs and fabrications of microheat exchangers [17].

2. Problem formulations

Consider a vertical annular microchannel of length l and width w, whose curvature radii of the inside and outside walls are κ_1 and κ_2, respectively. Both ends of the channel are open to the ambient. The inside wall of the channel is heated to a temperature greater than that of the surrounding gas, and the outside wall is maintained at the ambient temperature. Let r and z denote the usual cylindrical polar coordinates, let u_r and u_z denote the components of the velocity field, let the subscript 0 denote the ambient values, and let the subscripts 1 and 2 denote the inside-wall and outside-wall values, respectively. Under the usual Boussinesq approximation, we assume that the hydrodynamically fully developed conditions (zero streamwise velocity gradient, cross-flow velocity, and pressure-defect gradient: $\partial u_z/\partial z = 0$, $u_r = 0$, $\mathrm{d}\hat{p}/\mathrm{d}z = 0$) can be achieved after the fluid reaches the position $x = l^*$, and the nondimensionalized field equations can then be simplified as

$$\frac{1}{\eta+(1-\eta)R}\frac{\mathrm{d}}{\mathrm{d}R}\left((\eta+(1-\eta)R)\frac{\mathrm{d}U}{\mathrm{d}R}\right) = -\Theta, \tag{1}$$

$$\frac{1}{\eta+(1-\eta)R}\frac{\partial}{\partial R}\left((\eta+(1-\eta)R)\frac{\partial\Theta}{\partial R}\right) = RaU\frac{\partial\Theta}{\partial Z}. \tag{2}$$

It should be noted that the usual Boussinesq approximation is based on two principles. The first is that the variation of fluid density is considered only in the buoyancy term. The second is to assume that other physical properties were constant. Assuming a small temperature difference between the heated wall and the ambient supports the approximation [18]. Also, the effect of viscous dissipation for this low-speed flow and low-Prandtl-number fluid is negligible.

The nondimensionalized boundary conditions, which describe first-order velocity-slip and temperature-jump conditions at the fluid–wall interface, are

$$U(0) = \beta_v Kn\frac{\mathrm{d}U(0)}{\mathrm{d}R}, \tag{3}$$

$$U(1) = -\beta_v Kn\frac{\mathrm{d}U(1)}{\mathrm{d}R}, \tag{4}$$

$$\Theta(0,Z) = 1 + \beta_v KnIn\frac{\partial\Theta(0,Z)}{\partial R}, \tag{5}$$

$$\Theta(1,Z) = -\beta_v KnIn\frac{\partial\Theta(1,Z)}{\partial R}. \tag{6}$$

In Eqs. (1)–(6), we have used the following dimensionless parameters:

$$\left.\begin{array}{l} R = \frac{r-\kappa_1}{w}, \quad Z = \frac{z}{w}, \quad U = \frac{u_z}{u_c}, \quad \Theta = \frac{T-T_0}{T_1-T_0}, \quad u_c = \frac{\rho_0 g\beta_0(T_1-T_0)w^2}{\mu_0}, \\ \mathrm{Pr} = \frac{c_{p0}\mu_0}{k_0}, \quad Ra = \frac{\rho_0 c_{p0}u_c w}{k_0}, \quad Kn = \frac{\lambda}{w}, \quad In = \frac{\beta_t}{\beta_v}, \\ \eta = \frac{\kappa_1}{\kappa_2}, \quad \beta_v = \frac{2-\sigma_v}{\sigma_v}, \quad \beta_t = \frac{2-\sigma_t}{\sigma_t}\frac{2\gamma}{\gamma+1}\frac{1}{\mathrm{Pr}}, \quad \lambda = \frac{\sqrt{\pi\widehat{R}T_0/2}\mu_0}{p_0}, \end{array}\right\} \tag{7}$$

where T is the temperature, p is the pressure, g is the gravitational acceleration, ρ_0 is the density, μ_0 is the shear viscosity, c_{p0} is the constant-pressure specific heat, k_0 is the thermal conductivity, β_0 is the thermal expansion coefficient, λ is the molecular mean free path, $\widehat{R}$ is the specific gas constant, σ_v and σ_t are the tangential momentum and thermal accommodation coefficients, respectively, γ is the ratio of specific heats, η is the ratio of radii, Pr is the Prandtl number, Ra is the Rayleigh number, Kn is the Knudsen number, and In is the fluid–wall interaction parameter.

From Eq. (1), a solution in the form $U(Y)$ is only possible if Θ is a function of R only, i.e., $\partial\Theta/\partial Z = 0$. It implies that the flow under the assumption of hydrodynamically fully developed flow is also thermally fully developed. Eqs. (1) and (2) subject to the boundary conditions Eqs. (3)–(6) have the following analytical solutions:

$$\begin{aligned} U(R) = {} & B_0 + B_1\ln(\eta+(1-\eta)R) - \frac{A_0}{4(1-\eta)^2}(\eta+(1-\eta)R)^2 \\ & - \frac{A_1}{4(1-\eta)^2}((\eta+(1-\eta)R)^2\ln(\eta+(1-\eta)R) - (\eta+(1-\eta)R)^2), \end{aligned} \tag{8}$$

$$\Theta(R) = A_0 + A_1\ln(\eta+(1-\eta)R), \tag{9}$$

where

$$\left.\begin{array}{l} A_1 = \frac{-1}{\beta_v KnIn(1-\eta)(1+\frac{1}{\eta})-\ln\eta}, \\ A_0 = -\beta_v KnIn(1-\eta)A_1, \\ B_1 = \frac{\beta_v Kn(1-\eta)(2(\eta+1)A_0+(2\eta\ln\eta-\eta-1)A_1)+(\eta^2-1)A_0+(\eta^2\ln\eta-\eta^2+1)A_1}{4(1-\eta)^2(\beta_v Kn(1-\eta)(1+\frac{1}{\eta})-\ln\eta)}, \\ B_0 = \beta_v Kn\left(\frac{A_0}{2(1-\eta)} - \frac{A_1}{4(1-\eta)} - (1-\eta)B_1\right) + \frac{A_0}{4(1-\eta)^2} - \frac{A_1}{4(1-\eta)^2}. \end{array}\right\} \tag{10}$$

Three important parameters for convective gas microflow are the volume flow rate m, shear stress t^w, and local heat flux q. The dimensionless volume flow rate and the dimensionless shear stresses and local heat fluxes, expressed as Nusselt numbers, on the inside-wall and outside-wall surfaces of the microchannel are

$$\begin{aligned} M = {} & \frac{m}{2\pi w^2 u_c} = \frac{1}{(1-\eta)^2}\int_0^1(\eta+(1-\eta)R)U\,\mathrm{d}(\eta+(1-\eta)R) \\ = {} & \frac{B_0}{2(1-\eta)^2}(1-\eta^2) - \frac{B_1}{4(1-\eta)^2}(2\eta^2\ln\eta+1-\eta^2) \\ & - \frac{A_0}{16(1-\eta)^4}(1-\eta^4) + \frac{A_1}{64(1-\eta)^4}(4\eta^4\ln\eta+5-5\eta^4), \end{aligned} \tag{11}$$

$$T_1^w = \frac{t_1^w}{\mu_0 u_c/w} = \frac{\mathrm{d}U(0)}{\mathrm{d}R} = (1-\eta)\left(\frac{B_1}{\eta} - \frac{\eta}{2(1-\eta)^2}A_0 - \frac{2\eta\ln\eta-\eta}{4(1-\eta)^2}A_1\right), \tag{12}$$

$$T_2^w = \frac{t_2^w}{\mu_0 u_c/w} = -\frac{\mathrm{d}U(1)}{\mathrm{d}R} = (1-\eta)\left(-B_1 + \frac{A_0}{2(1-\eta)^2} - \frac{A_1}{4(1-\eta)^2}\right), \tag{13}$$

$$Nu_1 = \frac{q_1 w}{(T_1-T_0)k_0} = -\frac{\mathrm{d}\Theta(0)}{\mathrm{d}R} = \left(1-\frac{1}{\eta}\right)A_1, \tag{14}$$

$$Nu_2 = \frac{q_2 w}{(T_1-T_0)k_0} = -\frac{\mathrm{d}\Theta(1)}{\mathrm{d}R} = (\eta-1)A_1. \tag{15}$$

Here we have assumed that the channel walls are made of materials of low emissivity, so that the effect of thermal radiation is excluded.

3. Results and discussion

Air is used in most engineering application fields. Now we pay attention to the influence of wall-surface curvature on the microflow for air at the standard reference state. The physical properties at the chosen reference state can be found in Weng and Chen [18]. The present parametric study has been performed in the continuum and slip flow regimes ($Kn \leqslant 0.1$). Also, for air and various surfaces, the values of β_v and β_t range from near 1 to 1.667 and from near 1.64 to more than 10, respectively. So, this study has been performed over the reasonable ranges $0 \leqslant \beta_v Kn \leqslant 0.1$ and $1.64 \leqslant In \leqslant 10$ [3]. The chosen reference values of $\beta_v Kn$ and In for

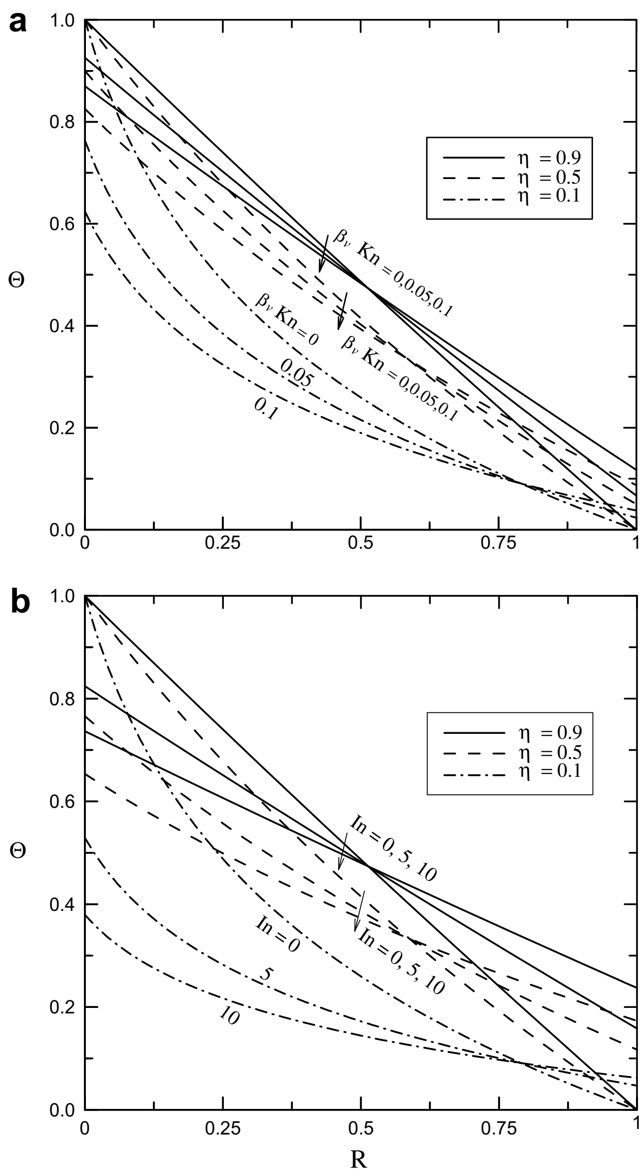

Fig. 1. Temperature distribution for (a) different values of $\beta_v Kn$ with $In = 1.64$, (b) different values of In with $\beta_v Kn = 0.05$.

the analysis are 0.05 and 1.64, respectively. The results with those for the macroscale case $\beta_v Kn = 0$ and with those for a physically impossible case $In = 0$ ($\sigma_t = 2$) are compared.

Natural convection in an annular microchannel can exhibit a nonlinear behavior in the temperature. The effect of decreasing the curvature radius ratio η is to raise the behavior, as shown in Fig. 1. In addition, it is clear that the increase in the rarefaction parameter $\beta_v Kn$ or the fluid–wall interaction parameter In leads to a larger temperature jump on the heated wall and a smaller temperature variation with R. As the value of η increases, both the effects increase. Fig. 2 shows that the increase in $\beta_v Kn$ leads to the increase in the velocity slip on the heated wall, but the increase in In leads to the decrease. The effect of decreasing η is to shift the velocity profile to the heated-wall side. In opposition to the slip induced by the rarefaction effect, the slip induced by fluid–wall interaction effect increases as η decreases.

Now we pay attention to the volume flow rate M, shear stresses T_1^w and T_2^w, and heat fluxes Nu_1 and Nu_2. Chen and Weng [3] has indicated that M is a monotone increasing function of $\beta_v Kn$. Moreover, In exerts no influence on M in parallel-plate microchannels. From Fig. 3, it is found that M is a monotone decreasing function of In. The fluid–wall interaction effect increases slightly with the decrease of the value of η, while the rarefaction effect decreases. From Fig. 4, it is obvious that at macroscale, the decrease in η leads to the increase in T_1^w and the decrease in T_2^w. Considering the heated wall, we find that as the value of $\beta_v Kn$ or In increases, the value of T_1^w decreases. Furthermore, such effects increase with the decrease of η. Under certain rarefaction and fluid–wall interaction conditions, it is possible to obtain reduced drag by decreasing the value of η. Recall that the physically impossible case $In = 0$ only provides the comparative base for the effect of fluid–wall interaction. We may, therefore, conclude from Fig. 5 that both Nu_1 and Nu_2 decrease with the increases of the values of $\beta_v Kn$ and In. It is interesting to note that as η decreases, Nu_1 increases but Nu_2 decreases. Moreover, the effects of $\beta_v Kn$ and In on Nu_1 increase, but the effects on Nu_2 decrease.

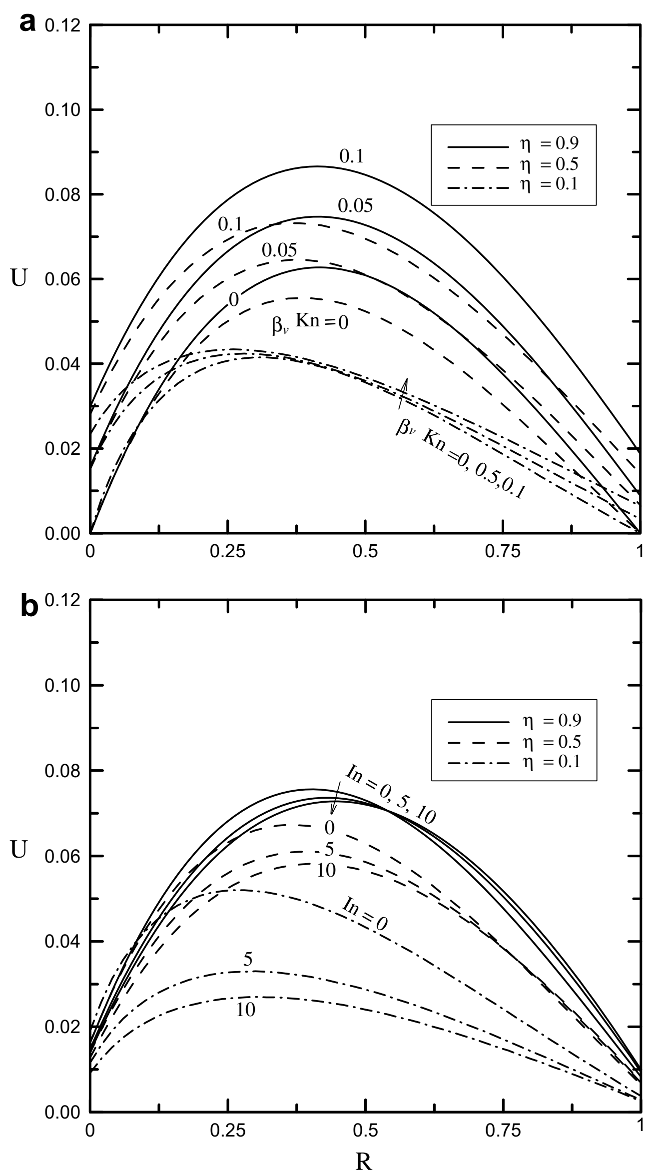

Fig. 2. Velocity distribution for (a) different values of $\beta_v Kn$ with $In = 1.64$, (b) different values of In with $\beta_v Kn = 0.05$.

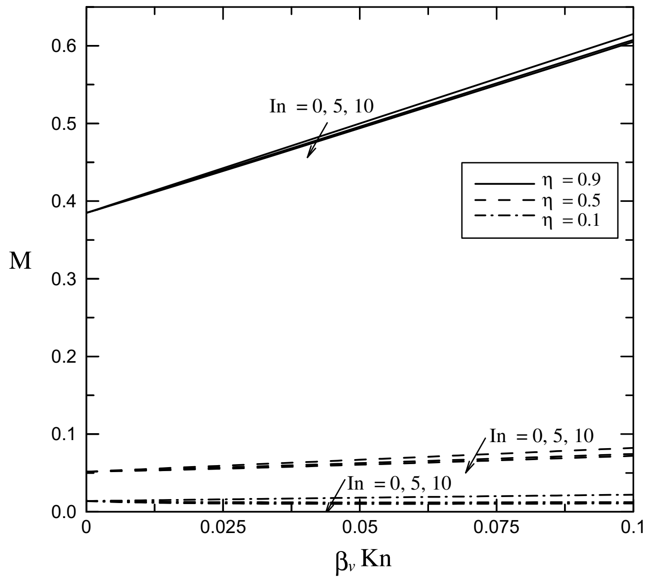

Fig. 3. Volume flow rate versus $\beta_v Kn$ for different values of *In*.

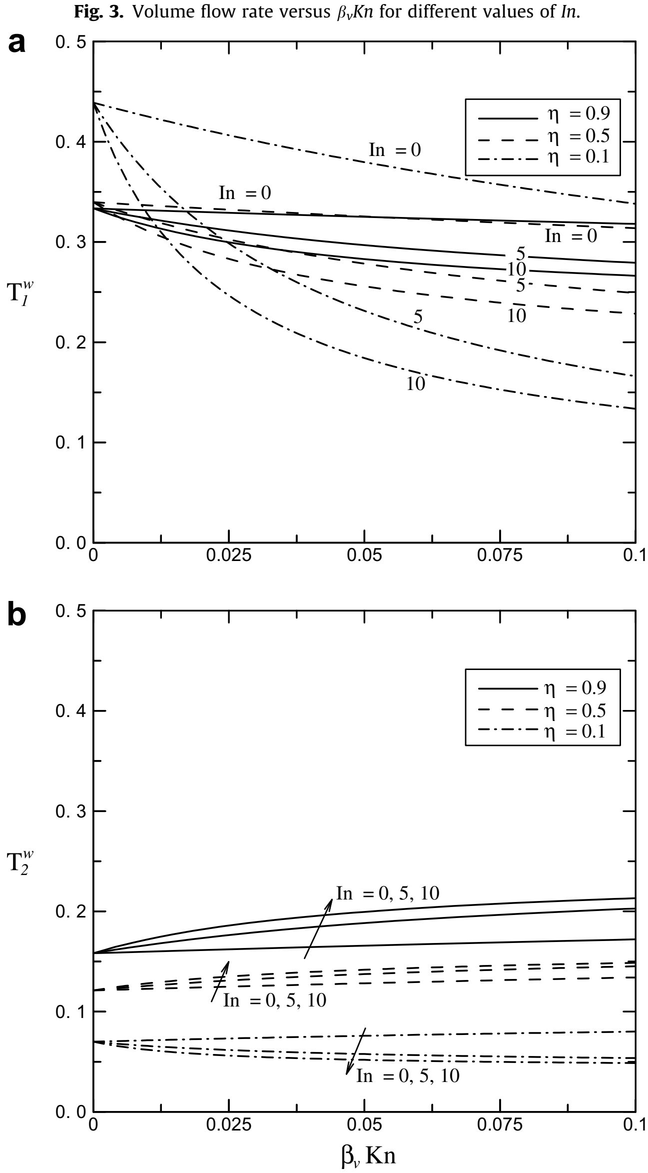

Fig. 4. (a and b) Shear stresses on the inside heated wall and outside unheated wall versus $\beta_v Kn$ for different values of *In*.

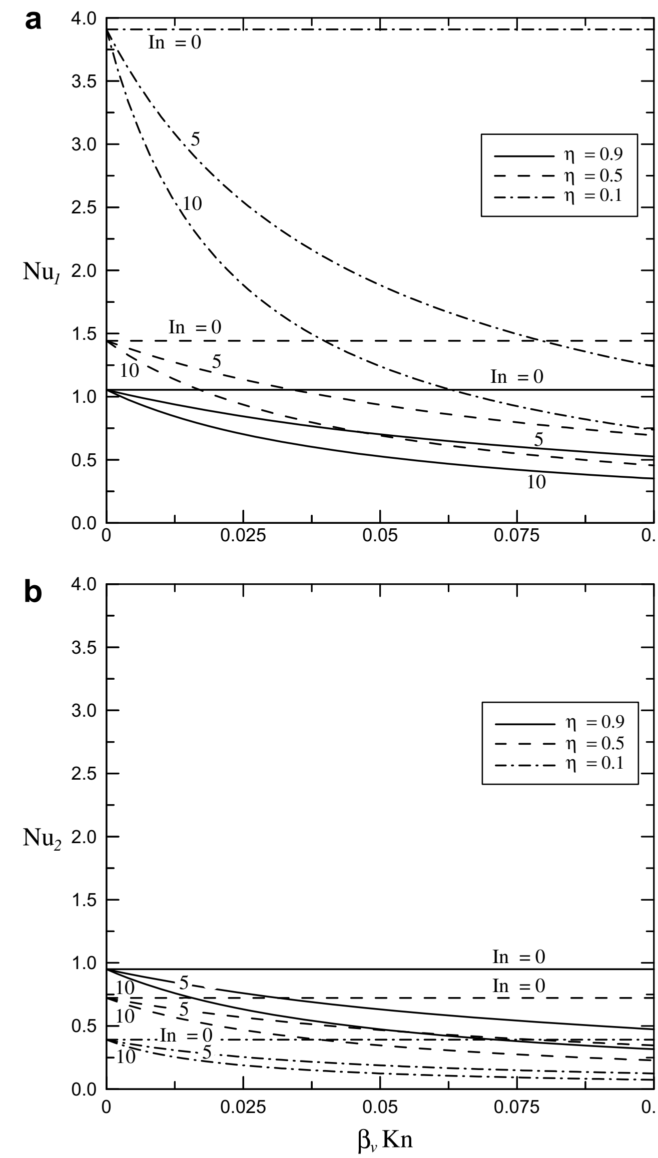

Fig. 5. (a and b) Heat fluxes on the inside heated wall and outside unheated wall versus $\beta_v Kn$ for different values of *In*.

4. Conclusions

An analytical study on the natural convection of gases over an isothermally heated wall of a vertical annular microchannel has been made. Along with the curvature radius ratio, the effects of rarefaction and the fluids–wall interaction were investigated. Results show that for this inside-wall cooling problem, the behavior of nonlinear temperature profile due to the curvature effect plays an important role in the flow drag and heat transfer. At macroscale, the decrease in the curvature radius ratio leads to the increases in the shear stress and heat flux on the heated wall. At microscale, the effects of rarefaction and fluid–wall interaction are to decrease the values of the shear stress and heat flux. As the value of the curvature radius ratio decreases, both the effects increase. Under certain rarefaction and fluid–wall interaction conditions, it is possible to obtain both drag reduction and heat transfer enhancement. In addition, the effect of rarefaction is shown to increase the flow rate, but the effect of fluid–wall interaction is shown to decrease it. Decreasing curvature radius ratio could block the rarefaction effect but promote the fluid–wall interaction effect.

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